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Review

Introduction

Parallelization Approaches

Parallelization Techniques

Synchronization

Summary

- Why are current processors increasing the core count instead of the clock rate?
 - 1. Higher clock rates require changing applications
 - 2. Increasing the clock rate also increases heat dissipation
 - 3. It is cheaper because cores can be interconnected more easily
 - 4. Additional cores increase memory throughput and graphics performance

- Which is the most-used architecture today?
 - 1. SISD: Single instruction stream, single data stream
 - 2. SIMD: Single instruction stream, multiple data streams
 - 3. MISD: Multiple instruction streams, single data stream
 - 4. MIMD: Multiple instruction streams, multiple data streams

- Which architecture requires explicit message passing?
 - 1. Shared memory
 - 2. Distributed memory
 - 3. Shared distributed memory
 - 4. Non-uniform memory access

- Which network topology requires only a single switch?
 - 1. Bus
 - 2. Ring
 - 3. Star
 - 4. Fat tree

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- Applications are traditionally written for serial execution
 - · Statements are turned into instructions by compiler/interpreter
 - · Instructions are executed serially by a single processor core
 - Only one instruction can be executed at a time
 - Instruction pointer (IP) indicates current instruction
- Performance is limited by clock rate of the single core
 - · Clock rate cannot be increased further due to heat issues
 - · Additional limitations due to memory and storage bandwidth

	Load
$IP \rightarrow$	Load
	Add
	Store

- Parallel applications execute instructions concurrently
 - · Problem has to be separated into concurrent parts
- · Parallel computers have multiple processing units
 - Allows working on problems concurrently
 - Can describe different resources: ALU, FPU, core etc.
- · Does not necessarily have to execute the same code
 - · Different applications can run at the same time





- · Memory access model as classifier
 - Determines programming model
- Shared memory
 - Processors consist of multiple cores
 - · Access to shared memory via a bus
 - · Limited scalability
- Distributed memory
 - · Processors only have access to own memory
 - · Machines are connected via a network
 - Better scalability



Introduction

Performance Development

- TOP500 list for supercomputers
 - Collected since the 1990s
- Exponential performance growth
 - Historically, factor 300-400 every ten years
 - Increase has slowed down significantly



- OpenMP is an interface for shared memory
 - · Applications run as multiple threads within a single process
 - OpenMP features thread management, task scheduling, synchronization and more
- MPI (Message Passing Interface) is an interface for distributed memory
 - · Applications run distributed over multiple compute nodes
 - MPI features message passing, input/output and other functions
- · Both approaches are available for multiple programming languages
 - In HPC, OpenMP and MPI are often used together

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- Parallelization consists of several aspects
 - 1. Take existing algorithm and try to make it run in parallel
 - 2. Come up with new algorithm that supports parallelism
 - 3. Implement the algorithm in a way that allows parallel execution
- · Application and data are distributed across resources
 - Related to SPMD and MPMD
- Different parallelization approaches
 - · Automatically, semi-automatically or manually

- Parallelization introduces additional overhead
 - · Either within in the application or the surrounding environment
 - · Some form of coordination is always required
- Aim for optimal use of resources
 - Using many components in parallel increases costs
 - · Optimal use is difficult to achieve, especially with overhead
- There are different kinds of overhead
 - · Additional computations are required due to distribution
 - · Partitioning the problem introduces more work
 - · Communication and synchronization are required to coordinate
 - Transformations for coupled applications

- · The most important architecture today is MIMD
 - SPMD and MPMD are high-level concepts that are often used on MIMD
- SPMD: Single program, multiple data streams
 - · All tasks execute same application but at different points
 - Application can use threads, message passing etc.
 - · Tasks use different data, for instance, using domain decomposition
 - There is typically logic to execute only parts of the application
 - For instance, coordination is performed by the first task

- SPMD distributes data across threads/processes
 - · Code is identical but can still perform different tasks
 - · Often used in combination with domain decomposition
 - For instance, two-dimensional matrix is the problem domain
- Decomposition is critical for achievable performance
 - Rows might be faster than columns depending on memory layout
 - Size of sub-domains determines load of each task
- Distribution also determines communication schema
 - Communication might have to be performed at boundaries

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0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15

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SPMD...



[Greenshields, 2016]

Michael Kuhn

Parallel Programming

- Benefits
 - Relatively easy to adapt to the available hardware
 - For example, increasing the matrix size allows using more main memory
 - · More tasks can be added by changing the decomposition slightly
 - Communication schemata are typically easy
 - Communication usually only happens at the sub-domain boundaries
 - · Debugging is much easier since only one program is involved
- Drawbacks
 - · Sometimes not appropriate for algorithm
 - · Load balacing might be difficult for dynamic problems

- MPMD: Multiple programs, multiple data streams
 - · Tasks execute different applications with different purposes
 - Application can use threads, message passing etc.
 - Tasks use different data, for instance, supplied by previous task
 - There is usually a functional decomposition
 - For instance, first task does pre-processing, last task does post-processing

- MPMD distributes functionality across processes/threads
 - Different code is distributed across tasks
 - Often used in combination with functional decomposition
 - For instance, chain of operations performed on pictures
- Not as common as SPMD due to specific requirements
 - Problem has to be able to be partitioned into multiple programs
 - · For instance, pre-process, calculation and finally post-process
- · Good fit for chains of operations
 - Compression, transformation, encryption etc.

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x' = f(x)

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- Benefits
 - · Appropriate for some widely available algorithms
 - Signal processing can run signal through multiple filters
- Drawbacks
 - Can be hard to tune for the available hardware
 - Requires more data to be available or functionality to be separated
 - Communication can become relatively complex
 - · Debugging is complicated since multiple programs have to be watched
 - · Balancing the computational load might also be complicated

- Some applications combine the SPMD and MPMD approaches
 - · Distributing data and functionality across threads/processes
- Climate models are a good example
 - · Climate can be seprated into different components
 - For instance, atmosphere, ocean, ice etc.
 - Each of of the components is too big to solve serially
 - Atmospheric data is distributed across tasks
 - Couplers are used to connect the components

- · Load distribution might be done statically or dynamically
 - · Load balancing means that we want to keep all tasks busy
- Static distribution is relatively easy
 - · Distribute data or loop iterations evenly if work is similar
 - This is often the case for numerical applications
 - Might result in load imbalance for varying computational work
 - For instance, particles migrate across domain boundaries
- Dynamic distribution requires more coordination
 - Might be done by a scheduler or a dedicated coordination task
 - · Results in better load balance for varying computational work

- What is the best way to distribute a matrix using SPMD?
 - 1. Each process holds one element
 - 2. Each process holds one row/column
 - 3. Each process holds several rows/columns
 - 4. Each process holds a sub-matrix

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- Different approaches for parallelization
 - · Automatic parallelization by the compiler
 - Semi-automatic parallelization by the compiler
 - Manual parallelization by the developer
- Technologies
 - Runtimes and libraries
 - Language extensions and new languages

- Automatic parallelization can be done by the compiler
 - There are approaches using Fortran etc.
 - · Compiler has to analyze data dependencies and determine feasibility
 - Compiler can then distribute data/loops/etc. across resources
- · Performance of existing solutions is usually not optimal
 - · Sometimes parallelization cannot be performed at all
- As with all automatic approaches, limited to particular patterns

- · Semi-automatic parallelization is supported by the compiler
 - · Developers have to identify opportunities for parallelization
 - Specifying compiler pragmas can give hints to the compiler
 - This may be combined with the automatic parallelization approach
- Most commonly used for shared memory
 - One popular example is OpenMP, which uses threads
 - Barriers, critical regions, atomic operations, reduction, tasks etc.
- There are also approaches for distributed memory
 - For instance, Chapel can distribute across multiple nodes

- · Manual parallelization puts the burden on the developer
 - Developers have to understand the problem at hand
 - Analyze algorithm for potential parallelism
- First step: Identify hotspots
 - Parallelize those first, since they require most computation
 - · If possible, use optimized software and libraries
- Identify bottlenecks
 - · Bottlenecks can limit performance if scaled up
- There are also algorithms that are hard to parallelize
 - One example is the Fibonacci sequence: f(n) = f(n-1) + f(n-2)
 - A possible solution is using another algorithm

- Runtimes can take care of distributing tasks intelligently
 - · For instance, submit many small tasks, runtime schedules them
 - This is often limited to the task level
- Libraries can support a wide range of use cases
 - · MPI offers communication and more for parallel programs
 - · POSIX Threads is a library-based approach for thread programming
 - Support for barriers, semaphors, mutexes, condition variables etc.

- Language extensions retrofit existing languages with support for parallelism
 - High Performance Fortran adds FORALL loops and more
 - C has native support for threads starting with C11
- New languages include parallelism into the core language design
 - Go has support for channels that can be used for parallelization
 - Rust can detect data races at compile time due to its ownership concept
 - Chapel, Erlang etc. have native support for distributed applications

- · Significant differences between numerical vs. non-numerical problems
 - Numerical: Weather, climate, fluid dynamics etc.
 - Non-numerical: Search engines, databases etc.
- Grand Challenges (US National Computing Research)
 - 1980: More funding for HPC in general
 - Computational fluid dynamics, electronic structure calculations, plasma dynamics, fundamental nature of matter, symbolic computations
 - 2000: Removing mostly completed research, adding new areas
 - Climate change, biological systems, virtual product design, cancer detection and therapy, modelling hazards

- Numerical problems are mostly iterative
 - For instance, simulations are often performed in time steps
 - Number of threads/processes is typically static
- Usually have global conditions for termination
 - In the easiest case, run for a specified number of time steps
 - · Alternatively, run until a condition is met
- Data structures are often regular
 - Data can often be stored in one or more matrices
 - Dimensionality of the matrices depends on the problem
 - Communication schemata are typically regular

- Many phenomena are highly parallel
 - Examples include galaxies, planets, climate and weather
- Many problems are very big or complex
 - Infeasible to solve them serially
 - Weather simulation has to be finished before it actually happens 🙂
- Parallel computing is well-suited
 - · Data and components can be distributed



[NOAA, 2007]

- · Parallelism also for non-numerical problems
 - Search algorithms, databases etc.
 - For instance, databases have to process many requests in parallel
- Some differences to numerical problems
 - · Speedup for tree searches depends on location
 - Parallelism might result in redundant work



[Grama et al., 2003]

- · Can this loop be parallelized automatically?
 - 1. Yes
 - 2. No, while loops cannot be parallelized
 - 3. No, there are dependency issues

```
1 while (true) {
2     c = calculate(c, ...);
3
4     if (c > x) {
5         break;
6     }
7 }
```

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Summary

- Processes are instances of an application
 - · Applications can be started multiple times
 - Processes are isolated from each other by the operating system for security reasons
 - · Resources like allocated memory, opened files etc. are managed per-process
- Threads are lightweight processes
 - · Threads have their own stacks but share all other resources
 - · Shared access to resources has to be synchronized
 - Uncoordinated access can lead to errors very easily
- · We will only take a look at threads for now
 - Message passing will be covered later

Process X					
Code, Memory, Files					
Thread 0 Thread 1 Thread 2					
Stack	Stack	Stack			
:	:	:			
•	•	•			

- Threads share a common address space
 - Communication is often done via shared variables
 - Threads are processed independently, that is, in parallel
 - If one thread crashes, the process crashes with all threads
- Processes have their own address spaces
 - Typically have to start multiple processes for distributed memory
 - · Overhead is normally higher than with shared memory
 - · There are also concepts for distributed shared memory
- In practice, hybrid approaches are used
 - A few processes per node (e. g., one per socket)
 - Many threads per process (e. g., one per core)

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 - Very convenient for users since no internals have to be known
 - · Reduced feature set in comparison to low-level approaches

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```
int main (void) {
       int i. iters = 0;
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3
       #pragma omp parallel for
4
       for (i = 0; i < 10000000; i++) {
5
6
            iters++;
7
        }
       printf("Iterations: %d\n", iters):
8
9
       return 0:
10
```

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2

3

4

5 6

7

8 9

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```
int main (void) {
    int i, iters = 0;
    #pragma omp parallel for
    for (i = 0; i < 100000000; i++) {
        iters++;
    }
    printf("Iterations: %d\n", iters);
    return 0;
}</pre>
```

```
$ time OMP_NUM_THREADS=1 ./openmp0
Iterations: 100000000
[...] 99% cpu 0,227 total
```

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$ time OMP_NUM_THREADS=1 ./openmp0
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```

```
$ time OMP_NUM_THREADS=2 ./openmp0
Iterations: 51147874
[...] 198% cpu 0,425 total
```

(or another number between 2 and 100,000,000)

- Parallel programming has at least two new error classes
 - 1. Deadlocks
 - 2. Race conditions
- A race condition has resulted in a wrong result in our example
 - Incrementing a variable consists of three operations
 - 1. Loading the variable
 - 2. Modifying the variable
 - 3. Storing the variable
 - · Operations have to be performed atomically

Debugging

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Parallel Programming

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T0	T1	V
Load 0		0
lnc 1		0
Store 1		1
	Load 1	1
	lnc 2	1
	Store 2	2

Parallel	programming	has at	least two	new	error o	classes
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	lnc 2	1
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Т0	T1	V
Load 0		0
Inc 1	Load 0	0
Store 1	lnc 1	1
	Store 1	1

Debugging...

- Deadlocks cause parallel applications to stop progressing
 - · Can have different causes, most often due to locking
 - May not be reproducible if there is time-dependent behavior
- · Error condition can be difficult to find
 - · Trying to lock an already acquired lock results in a deadlock
 - Erroneous communication patterns (everyone waits for the right neighbor)

Parallel Programming

- Error effect is typically easy to spot
 - · Spinlocks or livelocks can look like computation, though



- · Race conditions can lead to differing results
 - Debugging often hides race conditions
- · Error condition is often very hard to find
 - · Can be observed at runtime or be found by static analysis
 - Modern programming languages like Rust can detect data races
- Error effect is sometimes not observable
 - Slight variations in the results are not obvious
 - The correct result cannot be determined for complex applications
 - Repeating a calculation can be too costly

- critical protects an instruction or a scope with a lock
 - The locked part can only be entered by one thread at a time
 - It is possible to use atomic for simple instructions

2 3

4

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```
int main (void) {
    int i. iters = 0;
    #pragma omp parallel for
    for (i = 0; i < 10000000; i++) {
        #pragma omp critical
        iters++:
    }
    printf("Iterations: %d\n", iters);
    return 0;
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}
```

Thread 0	Thread 1	V
Load 0		0
Inc 1		0
Store 1		1
	Load 1	1
	lnc 2	1
	Store 2	2
:	•	:

2

3

4

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```
$ time OMP_NUM_THREADS=1 ./openmp1
Iterations: 100000000
[...] 99% cpu 1,464 total
```

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Iterations: 100000000
[...] 99% cpu 1,464 total
```

```
$ time OMP_NUM_THREADS=2 ./openmp1
Iterations: 100000000
[...] 194% cpu 6.615 total
```

- An alternative solution for the problem uses a reduction variable
 - Each thread has a separate private copy of the variable
 - At the end of the parallel region, all variables are reduced to one result

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 - · Each thread has a separate private copy of the variable
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int main (void) {
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        #pragma omp parallel for
4
             \hookrightarrow reduction(+:iters)
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        for (i = 0; i < 10000000; i++) {
             iters++:
6
 7
        }
8
        printf("Iterations: %d\n", iters);
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        return 0;
10
    }
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        printf("Iterations: %d\n", iters);
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        return 0;
10
```

V0	Thread 0	V1	Thread 1	V
0	Load 0	0	Load 0	
0	lnc 1	0	lnc 1	
1	Store 1	1	Store 1	
÷	:	÷	:	
50M	+	50M	=	100M

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            iters++:
        }
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        printf("Iterations: %d\n", iters);
9
        return 0;
10
```

```
$ time OMP_NUM_THREADS=1 ./openmp2
Iterations: 100000000
[...] 99% cpu 0,216 total
```

- An alternative solution for the problem uses a reduction variable
 - Each thread has a separate private copy of the variable
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int main (void) {
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        printf("Iterations: %d\n", iters);
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        return 0;
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```

```
$ time OMP_NUM_THREADS=1 ./openmp2
Iterations: 100000000
[...] 99% cpu 0,216 total
```

```
$ time OMP_NUM_THREADS=2 ./openmp2
Iterations: 100000000
[...] 197% cpu 0,106 total
```

- Why does the incorrect version get slower?
 - 1. Access conflicts on shared variable
 - 2. Increased memory traffic by threads
 - 3. Not slower because CPU time is measured

```
$ time OMP_NUM_THREADS=1 ./openmp0
Iterations: 100000000
[...] 99% cpu 0,227 total
```

\$	time	OMP_	_NUM_	THREAD)S=2 .	/openmp0
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- Parallel applications can execute instructions concurrently
 - Parallelization is necessary to solve complex problems
- SPMD and MPMD are high-level programming concepts
 - · SPMD distributes data cross tasks, while MPMD distributes functionality
- Parallelization can be done automatically, semi-automatically or manually
 - Compilers are not smart enough to do all the work for us (yet)
- Synchronization and communication are relevant on all abstraction levels
 - Real-world applications usually use hybrid approaches with MPI and OpenMP

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